Kawai-Kerman-McVoy Statistical Theory of Nuclear Reactions

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Fish Camp, CA

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Overview

- KKM theory derivation
- Numerical Tests of KKM
- KKM →KM
- Application to surrogate reactions

Feshbach's Projection Formalism (1962)

$P + Q = 1$

FKL (1967)

Intermediate (Doorway) Structure:
$P \rightarrow p + D$ D e.g. IAR

KKM (1973)

Direct channel coupling → Optical Background Rprsntn.

KM (1979)

Two-step reactions; surrogates
e.g. A(d,p)B*; B*→B+γ

FKK (1980)

$P \rightarrow P_1 + P_2 + P_3 + \ldots$
$Q \rightarrow Q_1 + Q_2 + Q_3 + \ldots$

Theory Summary

$$T_{cc'} = T_{cc'}^{(0)} + \frac{1}{2\pi}\sum_{\hat{q}} \frac{\hat{g}_{cq}\hat{g}_{c'q}}{E - \hat{\mathcal{E}}_q}$$

$$T_{cc'} = T_{cc'}^{\text{opt}} + \frac{1}{2\pi}\sum_{q} \frac{g_{cq}g_{c'q}}{E - \mathcal{E}_q}$$

KKM

$$\overline{\sigma_{cc'}^{\text{fl}}} \sim X_{cc}X_{c'c'} + X_{cc'}X_{c'c} \qquad X_{cc'} = \left\langle g_{cq}g_{c'q} \right\rangle_q$$

$$P_{cc'} = X_{cc'}Tr(X) + (X^2)_{cc'}$$

$$\overline{\sigma_{Rc}^{\text{fl}}} \sim X_{RR}X_{cc} + X_{Rc}X_{cR} \qquad X_{Rc} = \left\langle \mathcal{M}_{Rq}g_{cq} \right\rangle_q$$

KM

$$\mathcal{M}_{Rq} = M_R G_{\text{opt}} V_{Pq}$$

Projection operators

$$H\Psi = E\Psi$$

P – continuum space projection operator
Q – compound space projection operator

$$P + Q = 1 \;;\quad P \cdot Q = 0 \qquad P^2 = P \qquad H_{PQ} \equiv PHQ$$

$$\boxed{\begin{aligned}(E - H_{PP})P\Psi &= H_{PQ}Q\Psi \\ (E - H_{QQ})Q\Psi &= H_{QP}P\Psi\end{aligned}}$$

$$(E - H_{PP})\chi = 0$$

$$\Rightarrow \quad P\Psi = \chi + \frac{1}{E - H_{PP}} H_{PQ} Q\Psi$$

$$\Rightarrow \qquad T = T^{(0)} + \langle \chi | H_{PQ} | Q\Psi \rangle$$

$$(E - H_{QQ} - H_{QP} \frac{1}{E - H_{PP}} H_{PQ}) Q\Psi = H_{QP}\chi$$

$$\Rightarrow \qquad T = T^{(0)} + \langle \chi | H_{PQ} \frac{1}{E - H_{QQ} - H_{QP} \frac{1}{E - H_{PP}} H_{PQ}} H_{QP} | \chi \rangle$$

Projection operators cont'd.

$$T = T^{(0)} + \langle \chi | H_{PQ} \frac{1}{E - H_{QQ} - H_{QP} G_P H_{PQ}} H_{QP} | \chi \rangle$$

$$\left[H_{QQ} + H_{QP} G_P H_{PQ} \right] | \hat{q} \rangle = \hat{\mathcal{E}}_q \, | \hat{q} \rangle$$

$$\langle \tilde{\hat{q}} | \left[H_{QQ} + H_{QP} G_P H_{PQ} \right] = \langle \tilde{\hat{q}} | \, \hat{\mathcal{E}}_q$$

$$\hat{\mathcal{E}}_q = \hat{E}_q + i \frac{\hat{\Gamma}_q}{2} \qquad H_{QQ} | Q_j \rangle = E_{Q_j} | Q_j \rangle$$

$$\sum_{\hat{q}} | \hat{q} \rangle \langle \tilde{\hat{q}} | = 1 \qquad \sum_j | Q_j \rangle \langle Q_j | = 1$$

$$\langle \tilde{\hat{q}} | \hat{q}' \rangle = \delta_{\hat{q}\hat{q}'} \qquad \langle Q_j | Q_j \rangle = \delta_{ij}$$

$$T_{cc'} = T_{cc'}^{(0)} + \sum_{\hat{q}} \langle \chi_c | H_{PQ} | \hat{q} \rangle \frac{1}{E - \hat{\mathcal{E}}_q} \langle \tilde{\hat{q}} | H_{QP} | \chi_{c'} \rangle$$

$$T_{cc'} = T_{cc'}^{(0)} + \frac{1}{2\pi} \sum_{\hat{q}} \frac{\hat{g}_{cq} \hat{g}_{c'q}}{E - \hat{\mathcal{E}}_q}$$

KKM

Kawai, Kerman, and McVoy
Ann. of Phys. 75, 156 (1973)

$$(E - H_{PP})P\Psi = H_{PQ}\Psi \qquad (1)$$

$$(E - H_{QQ})Q\Psi = H_{QP}\Psi \qquad (2)$$

$$Q\Psi = \frac{1}{E - H_{QQ}} H_{QP}\Psi$$

$$(E - H_{PP} - H_{PQ}\frac{1}{E - H_{QQ}} H_{QP})P\Psi = 0 \qquad (3)$$

Lorentz. average, width I

$$(E - H_{PP} - H_{PQ}\frac{1}{E - H_{QQ} + iI} H_{QP})\overline{P\Psi} = 0$$

$$(E - H_{\text{opt}})\overline{P\Psi} = 0$$

Subtract H_{opt} from both Sides of (3)

$$(E - H_{opt})P\Psi = H_{PQ}\left(\frac{1}{E - H_{QQ}} - \frac{1}{E - H_{QQ} + iI}\right)H_{QP}\Psi$$

$$= \boxed{V_{PQ} G_Q V_{QP}} P\Psi$$

Two-pot. V_1, V_2

$$V_{PQ} \equiv H_{PQ}\sqrt{\frac{iI}{E - H_{QQ} + iI}}$$

$$\left(\frac{1}{E - H_{QQ}} - \frac{1}{E - H_{QQ} + iI}\right) = \frac{iI}{(E - H_{QQ})(E - H_{QQ} + iI)}$$

$$= \sqrt{\frac{iI}{E - H_{QQ} + iI}}\,\frac{1}{E - H_{QQ}}\,\sqrt{\frac{iI}{E - H_{QQ} + iI}}$$

KKM cont'd.

Separation of w.f. into average and fluctuating parts:

(also used in KM)

Used identities:

$(1-x)^{-1} = 1+x+x^2+\ldots$

$(AB)^{-1}=B^{-1}A^{-1}$

$X(1-YX)^{-1}=(1-XY)^{-1}X$

$$(E-H_{opt})P\Psi = V_{PQ}G_Q V_{QP}\Psi$$

$$\Rightarrow \quad P\Psi = \overline{P\Psi} + G_{\text{opt}}V_{PQ}G_Q V_{QP}\Psi$$

$$= \overline{P\Psi} + G_{\text{opt}}V_{PQ}G_Q V_{QP}\left[1+G_{\text{opt}}V_{PQ}G_Q V_{QP}+\ldots\right]\overline{P\Psi}$$

$$= \overline{P\Psi} + G_{\text{opt}}V_{PQ}G_Q V_{QP}\frac{1}{1-G_{\text{opt}}V_{PQ}G_Q V_{QP}}\overline{P\Psi}$$

$$= \overline{P\Psi} + G_{\text{opt}}V_{PQ}G_Q \frac{1}{1-V_{QP}G_{\text{opt}}V_{PQ}G_Q}V_{QP}\overline{P\Psi}$$

$$= \overline{P\Psi} + G_{\text{opt}}V_{PQ}\frac{1}{E-H_{QQ}-V_{QP}G_{\text{opt}}V_{PQ}}V_{QP}\overline{P\Psi}$$

KKM cont'd.

Two-potential formula:

$$T = T^{\mathrm{opt}} + \left\langle \overline{P\Psi} \middle| V_{PQ} \frac{1}{E - H_{QQ} - V_{QP} G_{\mathrm{opt}} V_{PQ}} V_{QP} \middle| \overline{P\Psi} \right\rangle$$

$$\left[H_{QQ} + V_{QP} G_{\mathrm{opt}} V_{PQ} \right] |q\rangle = \varepsilon_q \, |q\rangle$$

$$\langle \tilde{q} | \left[H_{QQ} + V_{QP} G_{\mathrm{opt}} V_{PQ} \right] = \langle \tilde{q} | \, \varepsilon_q$$

$$\varepsilon_q = E_q + i \frac{\Gamma_q}{2}$$

$$\sum_{\hat{q}} |q\rangle\langle\tilde{q}| = 1$$

$$\langle \tilde{q} | q' \rangle = \delta_{qq'}$$

$$|q\rangle = \sum_j \langle Q_j | q \rangle |Q_j\rangle$$

$$H_{QQ} |Q_j\rangle = E_{Q_j} |Q_j\rangle$$

$$\sum_j |Q_j\rangle\langle Q_j| = 1$$

$$\langle Q_j | Q_j \rangle = \delta_{ij}$$

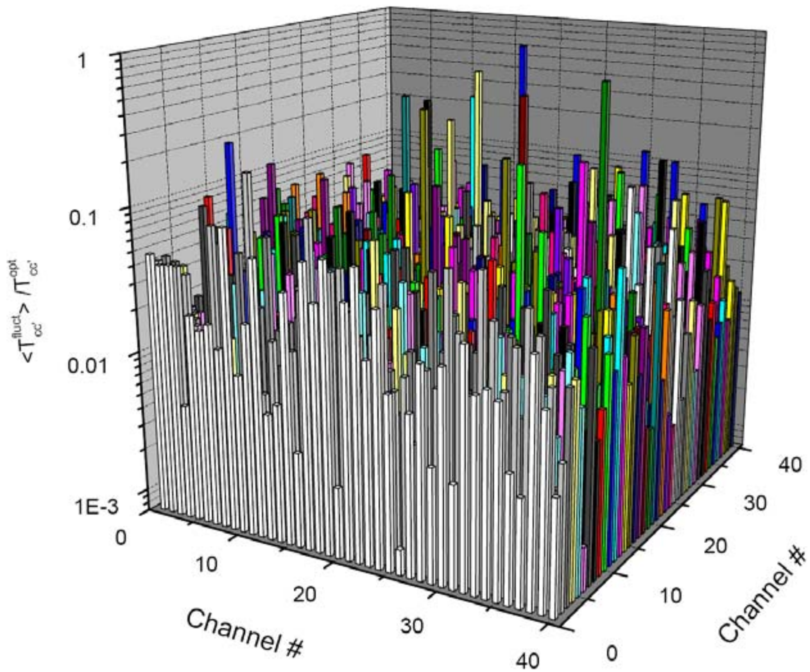
$$T_{cc'} = T_{cc'}^{\mathrm{opt}} + \sum_q \left\langle \overline{P\Psi_c} \middle| V_{PQ} \middle| \hat{q} \right\rangle \frac{1}{E - \varepsilon_q} \left\langle \tilde{\hat{q}} \middle| V_{QP} \middle| \overline{P\Psi_{c'}} \right\rangle$$

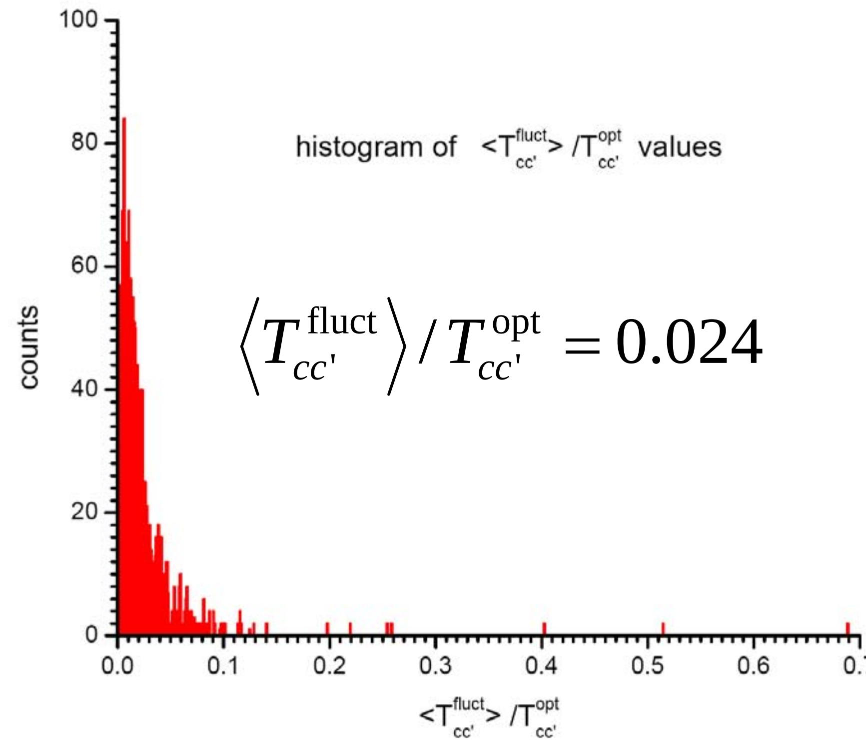
$$V_{PQ} = H_{PQ} \sqrt{\frac{iI}{E - H_{QQ} + iI}}$$

$$T_{cc'} = T_{cc'}^{\mathrm{opt}} + T_{cc'}^{\mathrm{fluct}} \quad , \quad T_{cc'}^{\mathrm{fluct}} \equiv \frac{1}{2\pi} \sum_q \frac{g_{cq} g_{c'q}}{E - \varepsilon_q} \Rightarrow \left\langle T_{cc'}^{\mathrm{fluct}} \right\rangle << T_{cc'}^{\mathrm{opt}} \quad \text{because} \quad \left\langle T_{cc'} \right\rangle \cong T_{cc'}^{\mathrm{opt}}$$

Verified numerically for Gaussian random coupling H_{PQ}.

Numerical Test of $\left\langle T_{cc'}^{\text{fluct}} \right\rangle / T_{cc'}^{\text{opt}} << 1$





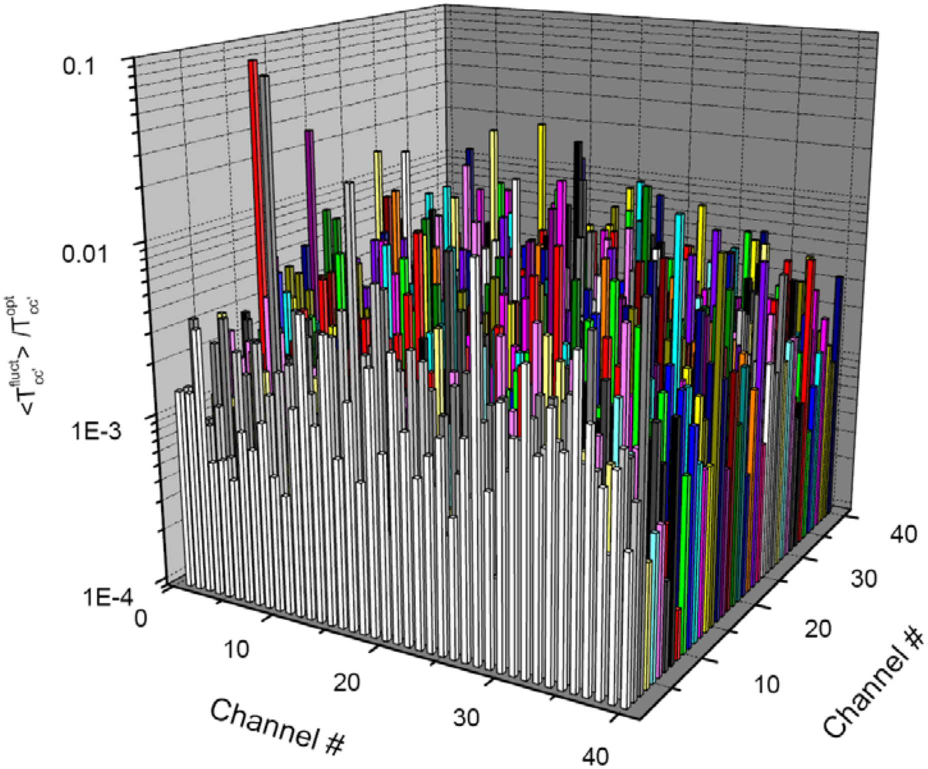
In the spirit of:
Dagdeviren and Kerman,
Ann. of Phys. **163** (1985) 199

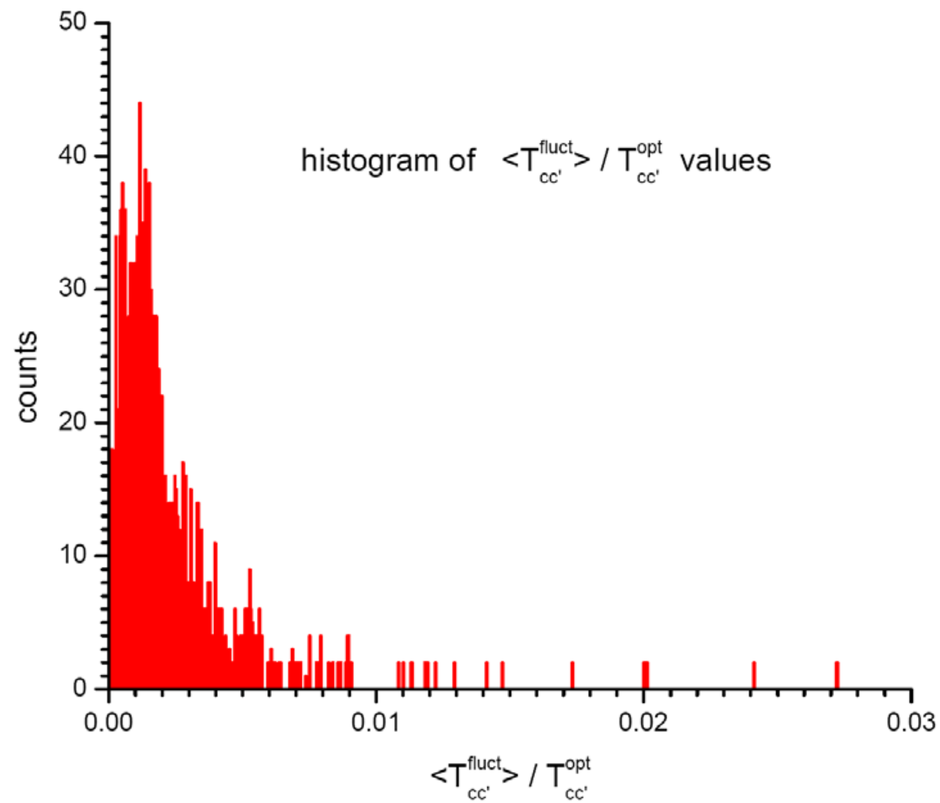
Computation parameters:

- 400 equidistant Q-levels
- 40 channels
- 20 equidistant radial points where H_{PQ} set to a Gaussian-distributed random interaction
- E = 20 MeV
- 100 E′ points for Lorentzian averaging between 18 and 22 MeV
- I = 0.5 MeV
- s-wave only
- $\Gamma/D \gg 1$

Cont'd. (incr. the # of Q-levels to 1600)

$$\left\langle \left\langle T_{cc'}^{\text{fluct}} \right\rangle / T_{cc'}^{\text{opt}} \right\rangle = 0.0024$$





KKM cont'd.

KKM Cross-section

$$T_{cc'} = T_{cc'}^{\mathrm{opt}} + \frac{1}{2\pi}\sum_q \frac{g_{qc}g_{qc'}}{E-\mathcal{E}_q}$$

$$\Rightarrow \langle \sigma_{cc'}^{\mathrm{fl}} \rangle \sim \left\langle \left| T_{cc'} - \overline{T}_{cc'} \right|^2 \right\rangle_I \sim \left\langle \sum_{qq'} \frac{g_{qc}g_{qc'}}{E-\mathcal{E}_q} \frac{g_{q'c}^* g_{q'c'}^*}{E-\mathcal{E}_{q'}^*} \right\rangle_I$$

Random Phase Hypothesis
→ only q=q' contributes

$$\cong \left\langle \sum_q \frac{g_{qc}g_{qc'}}{E-\mathcal{E}_q} \frac{g_{qc}^* g_{qc'}^*}{E-\mathcal{E}_{q'}^*} \right\rangle_I$$

$$\cong 2\pi \left\langle \frac{g_{qc}g_{qc'}g_{qc}^* g_{qc'}^*}{D_q \Gamma_q} \right\rangle_{q(I)}$$

$$\cong \frac{2\pi}{D_q \Gamma_q} \left\langle g_{qc}g_{qc'}g_{qc}^* g_{qc'}^* \right\rangle_{q(I)}$$

$$\cong X_{cc}X_{c'c'} + X_{cc'}X_{c'c} \qquad \text{where} \qquad X_{cc'} \equiv \left(\frac{2\pi}{D\Gamma}\right)^{1/2} \left\langle g_{qc} g_{qc'}^* \right\rangle_{q(I)}$$

Effect of KKM approximations could be studied numerically

See Appendix in KKM for details

KKM: Enhancement factor (σ/σ_{HF}) of KKM and Moldauer

(from Kawano, Bonneau, and Kerman, NDST 2007)

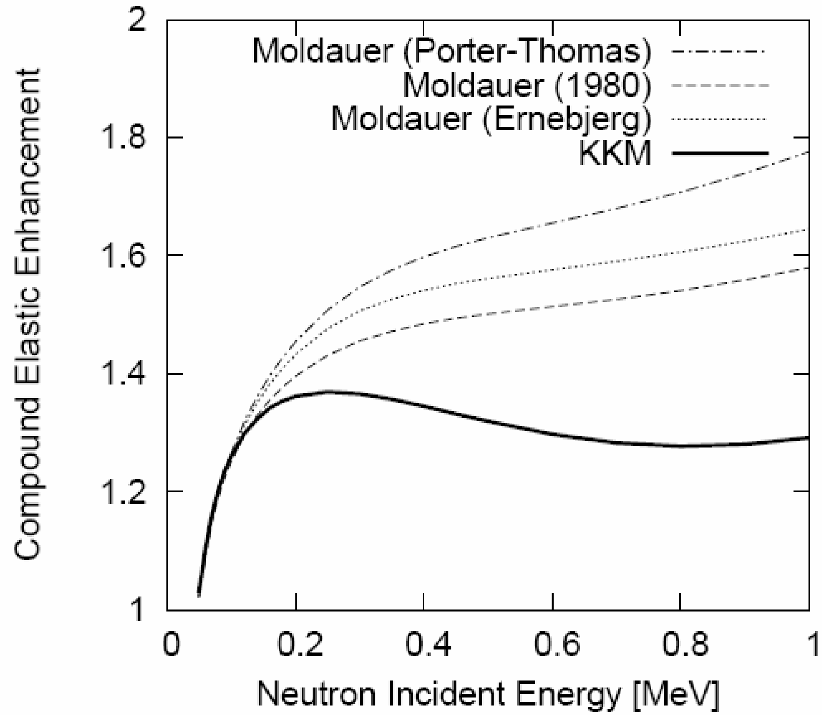


Fig. 2. Calculated compound elastic enhancement factors. The thick solid line is the KKM result, and the other lines are for the Moldauer calculations.

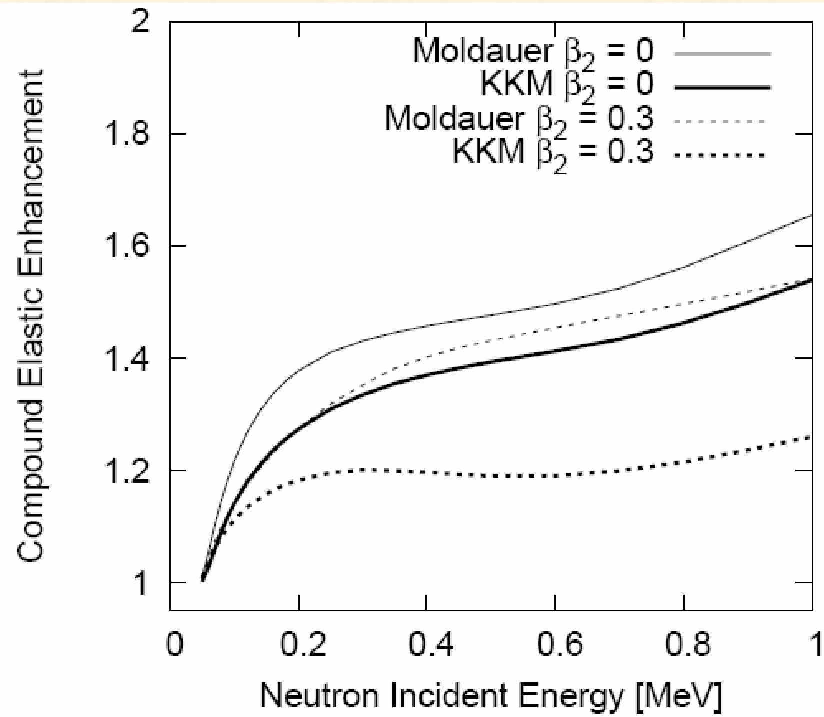


Fig. 3. Compound elastic enhancement factors for the spherical and strongly deformed cases. The thick lines are the KKM results, and the thin lines are for the Moldauer calculations.

CC on ^{238}U(n,n') ground state rotational band $0^+,2^+,4^+,6^+,8^+$

→ S-matrix → Transmission coefficients → X-matrix → σ_{KKM} vs. $\sigma_{cc'} = \frac{T_c T_{c'}}{\sum_a T_a} W_{cc'}$

The difference in U-238 <u>total</u> elastic cross-section is ~4%

KM T-matrix

$$T_{\mathrm{Rc}} = \left\langle \chi_{\mathrm{i}}^{(-)} \middle| M \middle| \chi_{\mathrm{f}}^{(+)} \Psi_c^{(+)} \right\rangle = M_R P \Psi_c^{(+)}$$

R=(i,f)

For example:
i – deuteron
f – proton
c - gamma

$$P\Psi_c = \overline{P\Psi}_c + G_{\mathrm{opt}} V_{PQ} \frac{1}{E - H_{QQ} - V_{QP} G_{\mathrm{opt}} V_{PQ}} V_{QP} \overline{P\Psi}_c$$

$$= \overline{P\Psi}_c + \sum_q G_{\mathrm{opt}} V_{Pq} \frac{1}{E - \mathcal{E}_q} V_{qP} \overline{P\Psi}_c$$

$$\Rightarrow \quad T_{\mathrm{Rc}} = T_{Rc}^{\mathrm{opt}} + \sum_q \frac{(M_R G_{\mathrm{opt}} V_{Pq}) g_{qc}}{E - \mathcal{E}_q}$$

$$= T_{Rc}^{\mathrm{opt}} + \sum_q \frac{\mathcal{M}_{Rq} g_{qc}}{E - \mathcal{E}_q}$$

$$= T_{Rc}^{\mathrm{opt}} + T_{Rc}^{\mathrm{fluct}}$$

KM fluctuation Cross-section

$$\left\langle \sigma_{Rc}^{\mathrm{fl}} \right\rangle \sim \left\langle \sum_{qq'} \frac{\mathcal{M}_{Rq} g_{qc}}{E-\mathcal{E}_q} \frac{\mathcal{M}_{Rq'}^{*} g_{q'c}^{*}}{E-\mathcal{E}_{q'}^{*}} \right\rangle_I$$

Random Phase Hypothesis

$$\cong \left\langle \sum_{q} \frac{\mathcal{M}_{Rq} g_{qc} \mathcal{M}_{Rq}^{*} g_{qc}^{*}}{(E-\mathcal{E}_q)(E-\mathcal{E}_q^{*})} \right\rangle_I$$

Analogous to KKM

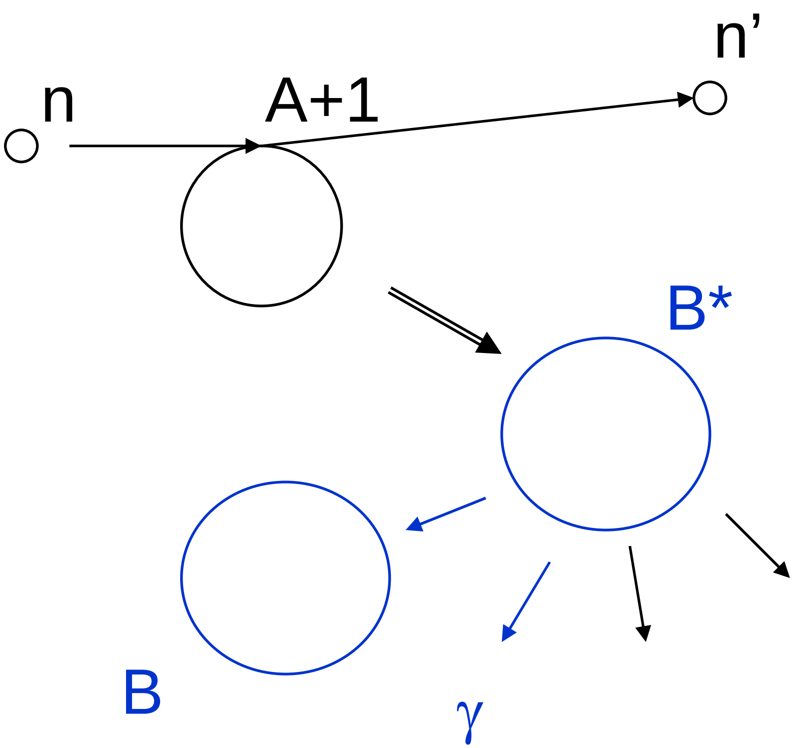
$$\cong 2\pi \left\langle \frac{\mathcal{M}_{Rq} g_{qc} \mathcal{M}_{Rq}^{*} g_{qc}^{*}}{D_q \Gamma_q} \right\rangle_{q(I)}$$

$$\cong \frac{2\pi}{D_q \Gamma_q} \left\langle \mathcal{M}_{Rq} g_{qc} \mathcal{M}_{Rq}^{*} g_{qc}^{*} \right\rangle_{q(I)}$$

$$\cong X_{RR} X_{cc} + X_{Rc} X_{cR} \qquad X_{RR} = \left\langle \mathcal{M}_{Rq} \mathcal{M}_{Rq}^{*} \right\rangle_{q(I)} \qquad X_{Rc} = \left\langle \mathcal{M}_{Rq} g_{qc}^{*} \right\rangle_{q(I)}$$

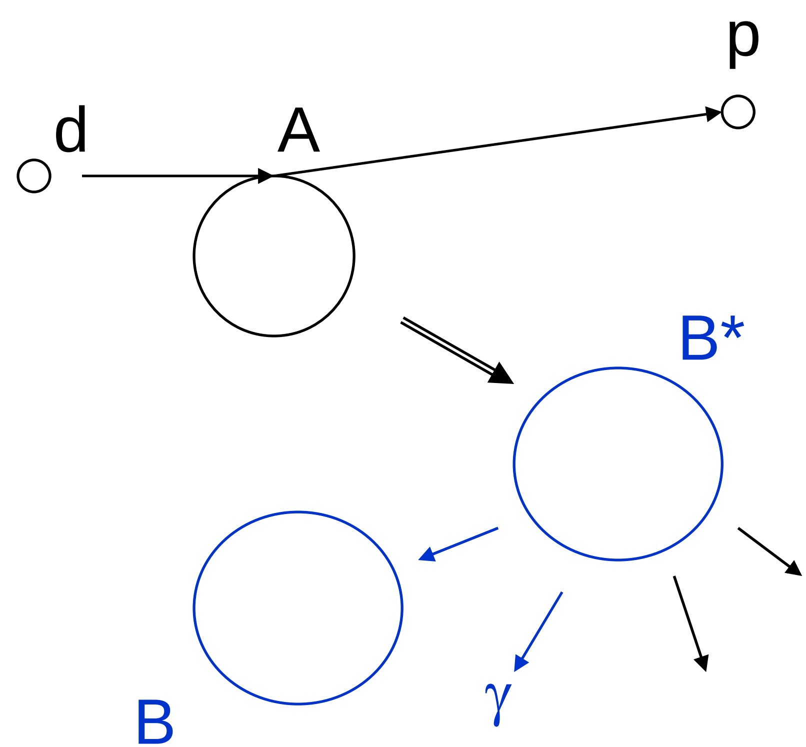
KM applied to Surrogate Reactions

"Desired" reaction, e.g. (n,n'γ)



$$\left\langle \sigma^{\mathrm{fl}}_{R'c} \right\rangle \cong X_{R'R'} X_{cc} + X_{R'c} X_{cR'}$$

Surrogate reaction, e.g. (d,pγ)



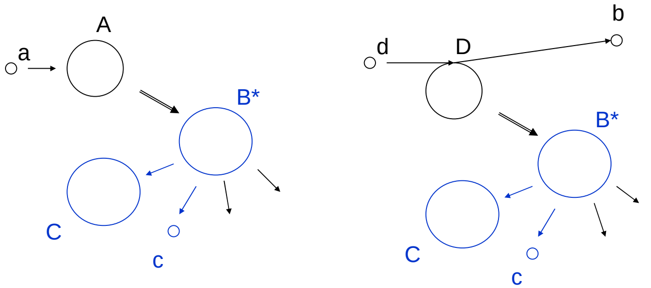
$$\left\langle \sigma^{\mathrm{fl}}_{Rc} \right\rangle \cong X_{RR} X_{cc} + X_{Rc} X_{cR}$$

Surrogate Reactions cont'd.

Desired reaction cross-section:

$$\frac{d\sigma^{\mathrm{HF}}_{\alpha\gamma}(E_\alpha)}{dE_\chi} = \sum_{J\pi} \sigma^{\mathrm{CN}}_{\alpha}(E_{\mathrm{ex}}, J, \pi)\, G^{\mathrm{CN}}_{\chi}(E_{\mathrm{ex}}, J, \pi)\;\; W$$

$$\alpha = (a + A)$$
$$\chi = (c + C)$$
$$\delta = (d + D)$$
$$E_{\mathrm{ex}} = S_a(B) + E_\alpha$$
$$= S_c(B) + E_\chi$$



Surrogate reaction probability:

$$P_{\delta\gamma}(E_{\mathrm{Ex}}) = \sum_{J\pi} F_\delta\,(E_{\mathrm{ex}}, J, \pi)\, G^{\mathrm{CN}}_{\chi}(E_{\mathrm{ex}}, J, \pi)$$

Jπ distributions are likely different for the two reactions:
complicates calculations and requires more surrogate data

J. Escher, F. Dietrich,
Phys. Rev. C74, 054601 (2006)

Conclusions and Outlook

- Extend Numerical Tests of KKM
  - Extend to Kerman-Sevgen theory
  - Look for connections to RMT and Max. Entropy methods
- KKM effects may be noticeable
  - dir. channel coupl. strong (Kawano et al.)
- Apply KM to surrogate reactions (LLNL collab.)
  - Inclusive reactions
  - Doorway states may play a role at intermediate resoloution

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