Absorption-Fluctuation Theorem for Nuclear Reactions: Brink-Axel, Incomplete Fusion and All That *

M. S. Hussein†[1,2]

[1] *Max-Planck-Institut für Physik komplexer Systeme*

Nöthnitzer Straße 38, D-01187 Dresden, Germany

[2] *Instituto de Física, Universidade de São Paulo*

C.P. 66318, 05315-970 São Paulo, S.P., Brazil

We discuss the connection between absorption, averages and fluctuations in nuclear reactions. The fluctuations in the entrance channel result in the compound nucleus,Hauser-Feshbach,cross section, the fluctuations in the intermediate channels, result in modifications of multistep reaction cross sections, while the fluctuations in the final channel result in hybrid cross sections that can be used to describe incoplete fusion reactions. We discuss the latter in details.

In the theory of nuclear reactions, one relies on statistical ideas to simplify their many-body nature. The first step is to introduce average amplitudes. The second step

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is to devise a Schrodinger equation that supplies these amplitues (the otical model). and the last step is the calculation of the fluctuation cross section. Several methods are available that supplies the derivation of this fluctuation cross section. The concept of energy averages and/or ensemble averages are used to obtain the Hauser-Feshbach cross section and corrections to it. The ergodic theorem is commonly employed to argue the equivalence of the two averaging methods. As much as fluctuations arise when one uses average amplitudes, the concept of fluctuations can be generalized to multistep processes, where the intermediate channel green functions are replaced by their averages. Similarly, if an exit doorway dominates the reaction, such as the excitation of a giant resonance in one of the nuclei participating in the reaction, or the coupling to the continuum, conveniently discretized, the final channel fluctuations set in resulting in an incomplete fusion cross section In both these cases, the absorption and fluctuation analysis is done on the wave exact entrance channel and exit channel wave functions.

In multistep reactions, another important reaction operator comes into the picture. The many-body intermediate channel Green's function. Again, one can decompose this quantity into an optical piece and a fluctuation piece. The fluctuation part can be calculated using the differenn optical quantitites as they appear in the KKM optical background representation. One important addition to this procedure is the inclusion of other collective states which might be excitated in the intermediate propagation of the system. We have in mind the Brink-Axel mechanism. This has been accomplished recently in the theoretical description of the excitation of multiphonon states. It is important to extend this new feature to multistep direct reactions in general.

In this talk I will discuss the incomplete fusion reaction theory and the Brink-Axel effect in multistep reaction theory. In both cases the Absorption- Fluctuation "theorem" is invoked to get a practical closed theory of the reactions.

The wave function fluctuation can be written as

$$\Psi = \Psi_{optical} + \Psi_{fluctuation}$$

where $\Psi_{fluctuation}$ can be related to the optical quantities contained in $\Psi = \Psi_{optical}$. By construction the energy average of the latter wave function is 0. This then results in an average cross section containing an optical one (calculated with DWBA or Coupled Channels Theory) and a fluctuation cross section calculated with optical transmission coefficients (or matrices). If the wave function refers to the final channel, then the resulting cross section will contain a direct one plus a fluctuation one which refers to the formation of a compound nucleus in a subsystem (if one has in mind a breakup process $a(b+x)+B \rightarrow b+(x+B) \rightarrow b+c+C$). The total formation cross section of the compound subsystem, $a(b+x)+B \rightarrow b+(x+B)$, is the incomplete fusion cross section.

When dealing with multistep processes, the intermediate channel Green's function, G_i can be decomposed into an optical plus a fluctuation pieces. The optical part contains complex energies. The fluctuation one is a multistep Green's function itself. It is this latter which would, if the conditions are met, contain the Brink-Axel effect.

Final State Fluctuation

Kerman & McVoy : Ann. Phys. 122, 197 (1979)
Hussein & McVoy : Nucl. Phys. A445, 124 (1985)
Austern et al. : Phys. Rep. 154 (1987) 125
Hussein, Frederico, Mastroleo, Nucl. Phys. A511, 269 (1990).

We consider a specific reaction

$$a(x+b) + A \longrightarrow b + \underbrace{(x+A)}_{\text{final channels}}$$

Consider b as a "spectator".

Cross section for observing b is (no CN formation in xA subsystem)

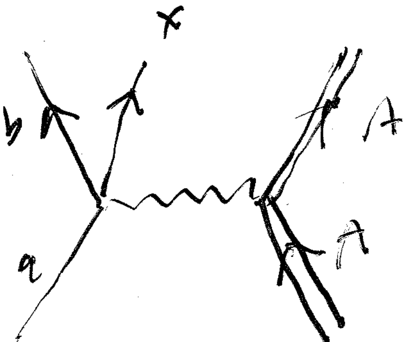
$$\frac{d^2\sigma^{dir.}}{d\Omega_b\, dE_b} = \frac{2}{\hbar v_a}\,\rho(E_b)\,\left\langle \hat{\rho}_x^{(+)} \right| \hat{W}_{XA}^{dir}(E_i + B_a - E_b) \left| \hat{\rho}_x^{(+)} \right\rangle$$

$$\rho(E_b) = \frac{\mu_b k_b}{(2\pi)^3\, \hbar^3}$$

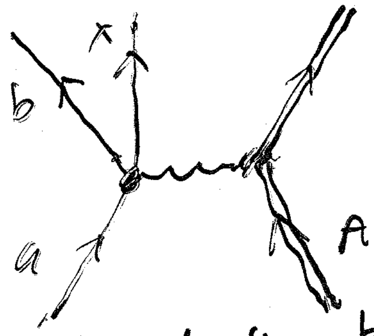
$$\hat{\rho}_x^{(+)}(\vec{r}_x) \equiv \left(\chi_b^{(-)}(\vec{r}_b) \right| \phi_a(\vec{r}_b - \vec{r}_x)\, \chi_a^{(+)}(\vec{r}_b, \vec{r}_x) \rangle$$

is the "negative-energy entrance channel" w.f. of x,

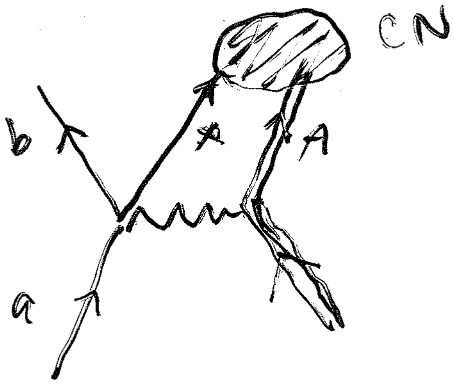
$$\frac{d^2\sigma^{di}}{d\Omega_b\, dE_b}$$



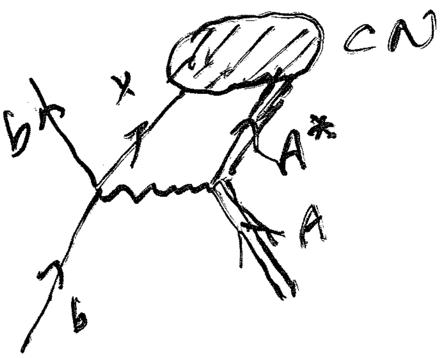
elastic break up



inelastic break-up



elastic break-up fusion



inelastic break up fusion

$$\frac{d^2\sigma^{fl}}{d\Omega_b\, dE_b}$$

See: Hussein and Mastroleo, Nucl. Phys. A 491 (1989) 468

If resonances are populated in the xA subsystem, need to calculate the fluctuation cross-section.

$$\frac{d^2\sigma^{fl}}{d\Omega_b\, dE_b} = \rho(E_b) \sum_c \frac{v_c}{v_a} \sigma^c_{capture}(E_i + B_a - E_b)$$

where

$$\sigma^c_{capture} \equiv \frac{2}{\hbar v_c} \langle \hat{\rho}_x^{(+)} | V_{P_0 P} [G_p^{(+)opt}]^\dagger W_p [G_p^{(+)opt}] V_{P P_0}$$

$$| \hat{\rho}_x^{(+)} \rangle$$

P_0 projects the "entrance" x-channel

At low energies, $\frac{d^2\sigma^{fl}}{d\Omega_b\, dE_b}$ becomes dominant

→ Hybrid Reactions : Direct, followed by Compound

<u>Names</u> : Incomplete Fusion, Surrogate Reactions, Trojan Horse Reactions etc.

General Characteristics of Hybrid Reactions :-

- "Incident" particle energy, E_x, is dispersed by an amount directly related to its internal (Fermi) motion in the projectile.

- The absorption into the CN proceeds in a multistep fashion;

$$|\hat{\rho}_c^{(+)}\rangle = G_p^{(+)\,opt}\, V_{p p_0}\, |\hat{\rho}_x^{(+)}\rangle$$

- Critical questions are not the CN decay ratios. Rather, it is the formation of the CN!

- the x particle could be a "phonon + photon" as in (α,α') reactions. How to treat these in view of the theory above.

Intermediate State Fluctuations

Carlson et al. Ann. Phys. 276, 111 (1999)
Gu & Weidenmüller, NPA 690 (2001) 382

When populating doorways in the final channel in a hybrid reaction and the doorways evolve into other collective states (giant resonances: 1 phonon → 2 phonons etc.), then the damping of the doorway into the background may, if the doorway is an intermediate channel, give rise to the excitation of a "doorway" on top of this background.

Brink-Axel resonances

Get more fluctuations!

factor of 2 to 3 smaller than the measured ones. A similar discrepancy, albeit somewhat smaller, is found for ^{208}Pb. This problem is a fundamental issue encountered in the double phonon excitation of complex Fermi systems.

Several effects that are not taken into account in the coupled-channel theory have been considered as possible explanations of this discrepancy. As examples, we mention the effect of anharmonicities [9, 10] and the quenching of the 1^+ DGDR state [11]. Here we will consider a new, potentially important mechanism, which consists in the (one-phonon) Coulomb excitation of background states responsible for the large spreading width of the one-phonon GDR, as suggested long ago by Brink and Axel [12]. Due to the complicated background of intrinsic states, the amplitude for this process varies rapidly wth energy and possesses an average close to zero. Its contribution to the cross section can be sizable, however. In close analogy to this situation is the well-known case of nucleon–nucleus elastic scattering. There, the cross section is the sum of the slowly varying contribution of average optical scattering and of the fluctuating contribution compound nucleus formation and decay. In Fig. 1 we show a schematic picture of the couplings involved.

We first sumarize our main result. The cross section for Coulomb excitation to the DGDR energy region contains in fact two distinct components which peak at $\sim 2E_{\mathrm{GDR}}$. However, while the usual component σ_{DGDR} has a width which may be estimated as $\sim 2\Gamma_{\mathrm{GDR}}$, the fluctuating Brink–Axel component has a width which is just $\sim \Gamma_{\mathrm{GDR}}$. As a result of this, the bump observed in the two-phonon region has an effective width between these limits. The enhancement factor for the peak value of the cross section will be given roughly by $(1+\Gamma_1^{\downarrow}/\Gamma_1)$, whereas the cross section

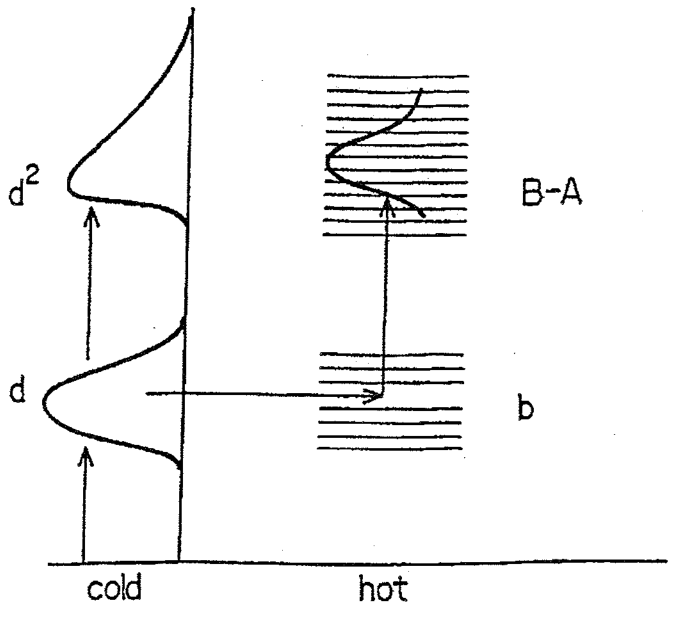


FIG. 1. A schematic picture showing the coupling to the one "cold" phonon state (d), the two "cold" phonon state, the fine structure states of d (b), and the one "hot" Brink–Axel (B–A) phonon state.

Amplitude for excitation of 1^{st} collective state

$$A_1(\vec{k}, \vec{k}_1') = \frac{1}{E - E_{k'} - e_1 + i\frac{\Gamma_1}{2}} \langle 1, \phi^{(-)}_{\vec{k}'} | V | \psi^{(+)}_{\vec{k}} \rangle$$

Energy of collective state $e_1 - i\frac{\Gamma_1}{2}$.

Corresponding cross section

$$\boxed{d\sigma_1(\vec{k}, \vec{k}') = \frac{\Gamma_1}{(E - E_{k'} - e_1)^2 + \left(\frac{\Gamma_1}{2}\right)^2} |\langle 1, \phi^{(-)}_{\vec{k}'} | V | \psi^{(+)}_{\vec{k}} \rangle|^2}$$

Second collective state is excited in a two-step process.

propagator in 1^{st} doorway subspace.

$$d\sigma_2(\vec{k}, \vec{k}'') = \frac{\Gamma_2}{(E - E_{k''} - e_2)^2 + \left(\frac{\Gamma_2}{2}\right)^2} |\langle 2, \phi^{(-)}_{k''} | V G^{(+)}_1 V | \psi^{(+)}_{\vec{k}} \rangle|$$

We can write $G^{(+)}_1 = \overline{G}^{(+)}_1 + G^{(+)}_1$ fl.

$$\overline{G}^{(+)}_1 = \frac{1}{E - E_{k'} - e_1 + i\Gamma_1/2}.$$

Then

$$d\sigma_2(\vec{k},\vec{k}'') = \overline{d\sigma_2(\vec{k},\vec{k}'')} + d\sigma_2^{fl}(\vec{k},\vec{k}'')$$

Where the intermediate-channel fluctuation cross-section is given by

$$\boxed{d\sigma_2^{fl}(\vec{k},\vec{k}'') \cong \frac{\Gamma_{BA}}{\left(E-E_{k''}-e_1-e_{BA}\right)^2+\left(\frac{\Gamma_{BA}}{2}\right)^2}\frac{\Gamma_1^{\downarrow}}{\Gamma_1} \left|\langle 1,\varphi^{(-)}_{k''}|\frac{V}{\sqrt{2}}\,\overline{G}_1^{(+)}\,V|\psi^{(+)}_{\vec{k}}\rangle\right|^2}$$

Excitation of 2nd collective state is then, taking $e_1+e_{BA}=e_2$

$$\boxed{d\sigma_2(\vec{k},\vec{k}'') = \left[\frac{\Gamma_{BA}/2}{\left(E-E_{k''}-e_2\right)^2+\left(\frac{\Gamma_{BA}}{2}\right)^2}\frac{\Gamma_1^{\downarrow}}{\Gamma_1}+\frac{\Gamma_2}{\left(E-E_{k''}-e_2\right)^2+\left(\frac{\Gamma_2}{2}\right)^2}\right] \times \left|\langle 1,\varphi^{(-)}_{k''}|V\,\overline{G}_1^{(+)}\,V\,|\psi^{(+)}_{\vec{k}}\rangle\right|^2}$$

Care must be taken when considering multistep processes involving giant resonances, as the Green function will exhibit the Brink-Axel fluctuation effect which may affect the convergence of the series!

Effect of Absorption on MSD Cross section

Amplitude : $\langle \psi_f^{(-)} | V + V G_j^{(+)} V + \cdots | \psi_i^{(+)} \rangle$

Use $G_j^{(+)} \longrightarrow G_{j,opt}^{(+)}$.

Second-order process :

$$V G_{i,opt}^{(+)} V \longrightarrow \int V \frac{|\varphi_{k''}^{(+)}\rangle \langle \tilde{\varphi}_{k''}^{(+)}|}{E_j - E_{k''} + i\varepsilon} V \, d\vec{k}''$$

FKK : take on-shell and get

$$V G_{i,opt}^{(+)} V \longrightarrow -i\pi V | \varphi_{k(E_j)}^{(+)} \rangle \langle \tilde{\varphi}_{k(E_j)}^{(+)} | V$$

Calculation with $\langle \tilde{\varphi}_{k_j}^{(+)} | V | \psi_{k_i}^{(+)} \rangle$ is orders (one) of magnitude larger than those with $\langle \varphi_{k_j}^{(-)} | V | \psi_{k_i}^{(+)} \rangle$

Reason

$$\langle \varphi_j^{(-)} | = S_j \langle \tilde{\varphi}_j^{(+)} |$$

$$|S_j| < 1$$

General question:

How to calculate $\langle \tilde{\varphi}_j^{(+)} |$?

Remember that the optical potential depends on energy and because of this it becomes complex (dispersive energy-dependence alá Feshbach)

Hussein and Moniz, Phys. Rev. C29 (1984) 2054

Consider the Schrödinger equation

$$(E - H(E))|\Psi^{(+)}\rangle = 0$$

Not true: $(E - H^{\dagger}(E))|\tilde{\Psi}^{(+)}\rangle = 0$ $\left[(E - H^{\dagger}(E))|\Psi^{(-)}\rangle = 0 \text{ correct!} \right]$

Because of energy-dependence in H, the dual state $\langle\tilde{\Psi}^{(+)}|$ is better defined through orthogonality

$$\langle \tilde{\Psi}^{(+)}_{\vec{k}'} | \Psi^{(+)}_{\vec{k}} \rangle = (2\pi)^3 \delta(\vec{k} - \vec{k}')$$

Write $\langle \vec{p} | \Psi^{(+)}_{\vec{k}} \rangle = (2\pi)^3 \delta(\vec{p} - \vec{k}) + \varphi^{(+)}_{\vec{k}}(\vec{p})$

$$\langle \tilde{\Psi}^{(+)}_{\vec{q}} | \vec{p} \rangle = (2\pi)^3 \delta(\vec{p} - \vec{q}) - \tilde{\varphi}^{(+)}_{\vec{q}}(\vec{p})$$

Thus, from orthogonality

$$\boxed{\tilde{\varphi}^{(+)}_{\vec{q}}(\vec{k}) = \varphi_{\vec{k}}(\vec{q}) - \int \frac{d\vec{p}}{(2\pi)^3} \varphi^{(+)}_{\vec{k}}(\vec{p})\, \tilde{\varphi}^{(+)}_{\vec{q}}(\vec{p})}$$

energy label momentum label.

Need to solve an integral equation to obtain $\tilde{\varphi}^{(+)}_{\vec{q}}(\vec{k})$!

Since $\varphi^{(+)}_{\vec{k}}(\vec{p}) = \langle \vec{p} | G_0^{(+)}(E_k)\, T(E_k) | \vec{k} \rangle$

$$= \langle \vec{p} | G^{(+)}(E_k)\, V(E_k) | \vec{k} \rangle ,$$

$$G^{(+)}(E_k) = (E_k - H_0 - V(E_k) + i\varepsilon)^{-1} ,$$

$$\tilde{\varphi}^{(+)}_{\vec{q}}(\vec{k}) = \langle \vec{q} | G^{(+)}(E_k) V(E_k) | \vec{k} \rangle$$

$$- \int \frac{d\vec{p}}{(2\pi)^3} \langle \vec{p} | G^{(+)}(E_k) V(E_k) | \vec{k} \rangle\, \tilde{\varphi}^{(+)}_{\vec{q}}(\vec{p})$$

$$\approx \langle \vec{q} | G^{(+)}(E_k) V(E_k) | \vec{k} \rangle$$

$$- \int \frac{d\vec{p}}{(2\pi)^3} \langle \vec{q} | G^{(+)}(E_p) V(E_p) | \vec{p} \rangle \langle \vec{p} | G^{(+)}(E_k) V(E_k) | \vec{k} \rangle$$

$$+ \cdots$$

If V does not depend on energy then

$$(E - H_0 - V^\dagger)\tilde{\psi}^{(+)} = 0$$

Appendix

The reaction $a(x+b) + A \rightarrow b + (x+A)$

Amplitude

$$T_{fi} = \langle \chi_b^{(-)} \psi_f^{(-)} | V_{xA}^{QP} | \phi_a \chi_a^{(+)} \Phi_0 \rangle$$

projectile target

$\psi_f^{(-)}$ is EXACT w.f. of the $(x+A)$ subsystem

Cross section

H_{xA}^{QQ}

$$\frac{d^2\sigma}{dE_b\, d\Omega_b} = \frac{2\pi}{\hbar v_a} \sum_f |T_{fi}|^2 \delta(E_b + E_f - B_a - E_i) \rho(E_b)$$

$$= \frac{2\pi}{\hbar v_a} \rho(E_b) \langle \chi_a^{(+)} \chi_b^{(-)\dagger} \phi_a \, \chi_0 | -\frac{\mathrm{Im}}{\pi} V_{xA}^{PQ} G_{xA}^{(+)} \cdot V_{xA}^{QP} | 0 \rangle \cdot$$

$$\cdot \, | \chi_n^{(+)} \chi_b^{(-)\dagger} \phi_a \rangle$$

Where

$$\hat{W}_{XA}(E_i + B_a - E_b) \equiv \langle 0 | - \frac{\mathrm{Im}}{\pi} V^{PQ}_{XA} G^{(+)}_{XA} V^{QP}_{XA} | 0 \rangle$$

Imaginary part of XA optical potential for x entering in the wave function $x_a^{(+)} \phi_a$ (namely <u>virtual</u>), rather than an a simple distorted wave.

If we have CN formation in XA then $G^{(+)}_{XA}$ must be further decomposed since instead of if H^{QQ}_{XA} we have now, after averaging the CN state

$$G^{(+)}_{XA} \longrightarrow \overbrace{\frac{1}{E_x - H_{opt} + i\varepsilon}}^{G^{(+)}_{XA;opt}} + G^{(+)}_{XA}\text{ fl.}$$

averages out to zero

$$\mathrm{Im}\, G^{(+)}_{XA;opt} = -\pi \sum_c |\varphi_c^{(-)}\rangle\langle\varphi_c^{(-)}|\, \delta(\quad)$$

$$+\, G^{(+)\dagger}_{opt}\ \mathrm{Im}\, H_{opt}\, G^{(+)}_{opt}.$$

$$(E_x - H_{opt})|\phi_c^{(-)}\rangle = 0$$

Conclusions

- **Hybrid reactions**

Important activity for theorists would be to find out why the Surrogate method works so well. Find out cases where it does not work

- **Multistep reactions**

- Elaborate more on intermediate state fluctuations, Brink-Axel, etc.
- Analyse the energy-dependence of the optical potential and calculate more precisely the dual state scattering wave function